Cristian derivation

In Fourier space:

my work

if for a point charge interacting with a anisotropic surface the potential is

then for a second point charge in the same system can be expressed as

where

is the displacement of the second particle in the z direction is the displacement in the R direction, is the angel between the arbitrary point and the second point charge in the R plain at the first point charge

using the principle of superposition, the potential at an arbitrary point is given by

Diagram

Description automatically generated

determining with and (where is the angle between the dipole and the point and is the angle between the dipole and the normal to the interface)

let

using Taylor expansion around d=0 and with and higher terms being ignored

therefore the first two terms of these expressions gooses to

substituting the Taylor expansions in we get

as

test cases

returns the expected result for a dipole in a single medium. as is the distance form the point to the first point charge squared

returns the expected result for a dipole at an isotropic interface.

d=0

as expected for two oppositely charged point charges on top of one another

when compared to the case for an non hyperbolic metamaterial we see that the image charge screening factor needs to be modified by

implementing this in to the 4 layer stack model as below this is for sheets the only difference between this and the nanowires version is how are calculated (the methods are given in ‘Hyperbolic metamaterials: fundamentals and applications’)

electric permittivity given by Drude-Lorentz(DL) model:

isotropic polarizability of each free standing NP (using quasi-static dipolar approximation)

as half space is anisotropic then the dielectric permittivity perpendicular and parallel to the interface are different given by

due to light being comprised of s and p polarised light then the quasistatic polarizability can be expressed as

the image-charge screening factor for the film is given as (determined from potential derivation)

the lattice dependents parameters for a hexagonal lattice are given by

meaning that for the NP layer the parallel and perpendicular components correspond to

from the transfer matrix the reflection and transmittance can be calculated by

so

the transmittance requires a scaling factor however this is not reliable due to layer 4 being hyperbolic which requires more info the one here is the general one for isotropic L4 (non metal) so transmittance is ignored in all plots this can be included in a 5 layer system

then

where: